Implementation of Speed Sensor-less Field Oriented Control for Permanent Magnet Synchronous Motor (Surface and Interior)

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1 Introduction

Permanent Magnet Synchronous Motors (PMSMs) are being used in wide applications, mainly due to high energy density and robustness. Two variants of PMSM exist, depending on how the rotor is constructed:

1. Surface mount: The magnets are mounted on the surface of the rotor. They need special profiling to get sinusoidal BEMF. Due to symmetry in the airgap due to this profiling, the magnetic saliency of inductance along the magnet and in quadrature to magnet is negligible.

2. Interior mount: The rotor is cylindrical, having even stresses throughout. Permanent bar magnets are put deep inside the rotor. Due to the asymmetry in airgap caused due to bar magnets, the magnetic saliency of inductance along the magnet and in quadrature to magnet is significant. But, since this saliency gives rise to a ‘reluctance torque’ (like switched reluctance motor – SRM), the additional torque can be exploited to achieve higher efficiency.

This document describes the algorithms developed for PMSM driven loads. An example for load on SPMSM is a washing machine, while a load on IPMSM load is a compressor. The algorithm is based on FOC of PMSM with AN1292 as baseline, but changed significantly to incorporate specific needs of both SPMSM (henceforth, PMSM and SPMSM are used interchangeably) and IPMSM based drive.

Load specific algorithms like Torque Compensation (Compressor); Wind-milling, Initial Position Detection (Fan); Load Balancing (Washing Machine) is out of scope for this document.

1.1 Nomenclature

This section deals with list of symbols, constants etc. used in this document.

1.1.1 Motor Parameters

Motor related parameters are listed below:

r_s	Stator Resistance
L_{ds}	D-axis Stator Inductance
L_{qs}	Q-axis Stator Inductance
Ψ_{PM}	Motor Back EMF (BEMF) constant
V_α	Applied voltage along phase 'a'
V_β	Applied voltage quadrature to phase 'a'
I_α	Stator current along phase 'a'
I_β	Stator current quadrature to phase 'a'
E_α	BEMF along phase 'a'
E_β	BEMF quadrature to phase 'a'
ω	Rotor speed
θ	Estimated BEMF angle
θ_0	Actual BEMF angle
v_{ds}	Applied voltage along d-axis
v_{qs}	Applied voltage along q-axis
I_{ds}	Stator current along d-axis
I_{qs}	Stator current along q-axis
V_{max}	Maximum applied voltage magnitude
V_{dc}	Sensed dc link voltage

1.1.2 Control Parameters

Control related parameters are listed below:

thetaOL	Open Loop Angle
velRef	Speed reference
IdRef	D-axis current reference
IqRef	Q-axis current reference
Startup_Lock	Startup locking time counter
Startup_ramp	Startup angle reference accumulation counter - scaled
spd_ramp_count	Speed slew rate counter
ThetaOffset	Difference between estimated and open loop angle
EstimTheta	Estimated rotor angle
OmegaFilt	Speed feedback filtered
maxSpdVdc	Nominal speed calculated at a given dc link voltage
dead_time	Dead Time
PWM_period	PWM Period
Invkfi	Inverse of BEMF constant
PINBUTTON1	Pushbutton for starting or stopping of motor
VdcVal	DC Link Voltage
stop_cmd	Stop Command to motor
V_{dc}	Sensed dc link voltage
OffsetCalc	Current offset calculated flag

1.1.3 Accumulation Variables

Accumulation Variables are listed below:

PIParmD.qdSum	D Axis Integrator Accumulator
PIParmQ.qdSum	Q Axis Integrator Accumulator
PIParmQref.qdSum	Speed loop Integrator Accumulator
Startup_Ramp	Startup angle reference accumulator counter - scaled
spd_ramp_count	Speed slew rate counter
OmegaFiltStateVar	Speed Filter accumulator
ThetaStateVar	Estimated angle accumulator
KiOut	Integrator accumulator of estimator
OmegaPIOutFiltTemp	PI output filter accumulator of estimator
EdFiltStateVar	Ed filter accumulator
EqFiltStateVar	Eq filter accumulator
ThetaErrorAcc	Angle Error accumulator
countOpenLoop	Counter for angle error accumulation
min_time_cntr	Minimum time in braking state counter
curr_hold_cntr	Current held in limits during braking counter

1.1.4 Constants

Constants and their values are listed below:

Constant	Value	Comment
MIN_DC_VOLT_MTR_RUN		Minimum dc link voltage needed to run the motor: Board dependent
FW_SPD_THD_GAIN	0.8	Factor for nominal speed calculation after considering variations
PI_BY_2	π/2	90 deg
PI_BY_3	π/3	60 deg
PI_BY_6	π/6	30 deg
VMAX_SQ	0.98 pu	Maximum modulation index magnitude
Motor Dependent Constants		
ENDSPEED		Transition speed scaled for open loop functioning
ENDSPEED_ELECTR		Transition speed scaled in Electrical RPM
Q_CURRENT_REF_LOCK		Locking current reference
Q_CURR_REF_OPENLOOP		Open loop current reference
OPENLOOP_SPD_RATE		Speed slew rate
OPENLOOP_SPD_COUNT_LIMIT		To reduce the speed slew rate
FD_WEAK_IDREF_LT		Maximum flux weakening current reference
END_SPEED_RPM		Transition speed to close loop
LOCK_TIME		Time given for rotor locking
D_CURRCNTR_PTERM		Kp for D axis Current Loop
D_CURRCNTR_KPSCALE		Scaling for Kp of D axis Current Loop
Q_CURRCNTR_PTERM		Kp for Q axis Current Loop
Q_CURRCNTR_KPSCALE		Scaling for Kp of Q axis Current Loop
D_CURRCNTR_ITERM		Ki for D axis Current Loop
Q_CURRCNTR_ITERM		Ki for Q axis Current Loop
SPEEDCNTR_PTERM		Kp for Outer Speed Loop
MTR_TIME_CONST_COUNTS		Motor time constant
IPD_PULSE_ON_TIME		ON time for pulses during IPD
IPD_PULSE_OFF_TIME		OFF time for pulses during IPD
WINDMILL_MIN_SPD		Minimum speed for windmill
WINDMILL_MAX_SPD		Maximum speed for windmill
WINDMILL_MIN_BEMF		Minimum BEMF to detect windmill
WINDMILL_CHK_CNT_MAX		Counts to check windmilling is happening or not
WINDMILL_ZERO_CRS_MIN		Zero cross for BEMF - lower threshold
WINDMILL_ZERO_CRS_MAX		Zero cross for BEMF - upper threshold
WINDMILL_FREQ_NUMERATOR		Numerator to calculate windmilling frequency
WINDMILL_ANGLE_OFFSET		Angle offset while closing the loop from windmilling

2 Specifications and Requirements

Following are specifications and Requirements for this compressor:

1. Speed Sensor less drive
2. No Lookup Tables to be included in the algorithm
3. Include MTPA for maximum efficiency where applicable

3 Main State Machine

The main state machine which controls motor modes is described below:

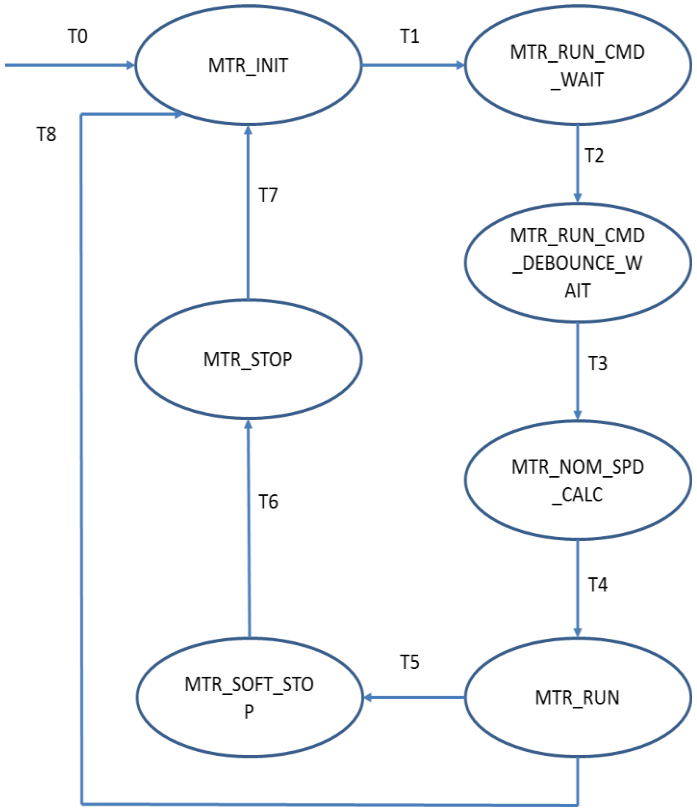


Figure 3.1: Main State Machine

3.1 State Description

Main state description is as follows:

S.No.	State	Description
1	MTR_INIT	Motor run time initializations
2	MTR_RUN_CMD_WAIT	Waits for run command
3	MTR_RUN_CMD_DEBOUNCE_WAIT	Waits for debouncing of run command
4	MTR_NOM_SPD_CALC	Calculates nominal speed based on dc link voltage
5	MTR_RUN	Run the motor
6	MTR_SOFT_STOP	Slowly decrease speed of motor for safe stopping
7	MTR_STOP	Motor stops

4 Field Oriented Control

In this type of control, the controlling action is performed in a rotating reference frame aligned with rotor flux. In such a reference frame, the sinusoidal currents become DC and hence, a PI can regulate them to their set point.

By aligning the reference frame to rotor flux, torque and flux producing components of currents are decoupled and can be controlled independently. At speeds below nominal speeds, since the entire flux is produced by rotor, the flux producing component can be made zero (SPMSM) or negative to aid permanent magnet torque (IPMSM), thus improving the efficiency of the motor.

This algorithm is being run when the main state machine is in either MTR_RUN or MTR_SOFT_STOP state

4.1 FOC Block Diagram

The following block diagram depicts complete control scheme of FOC:

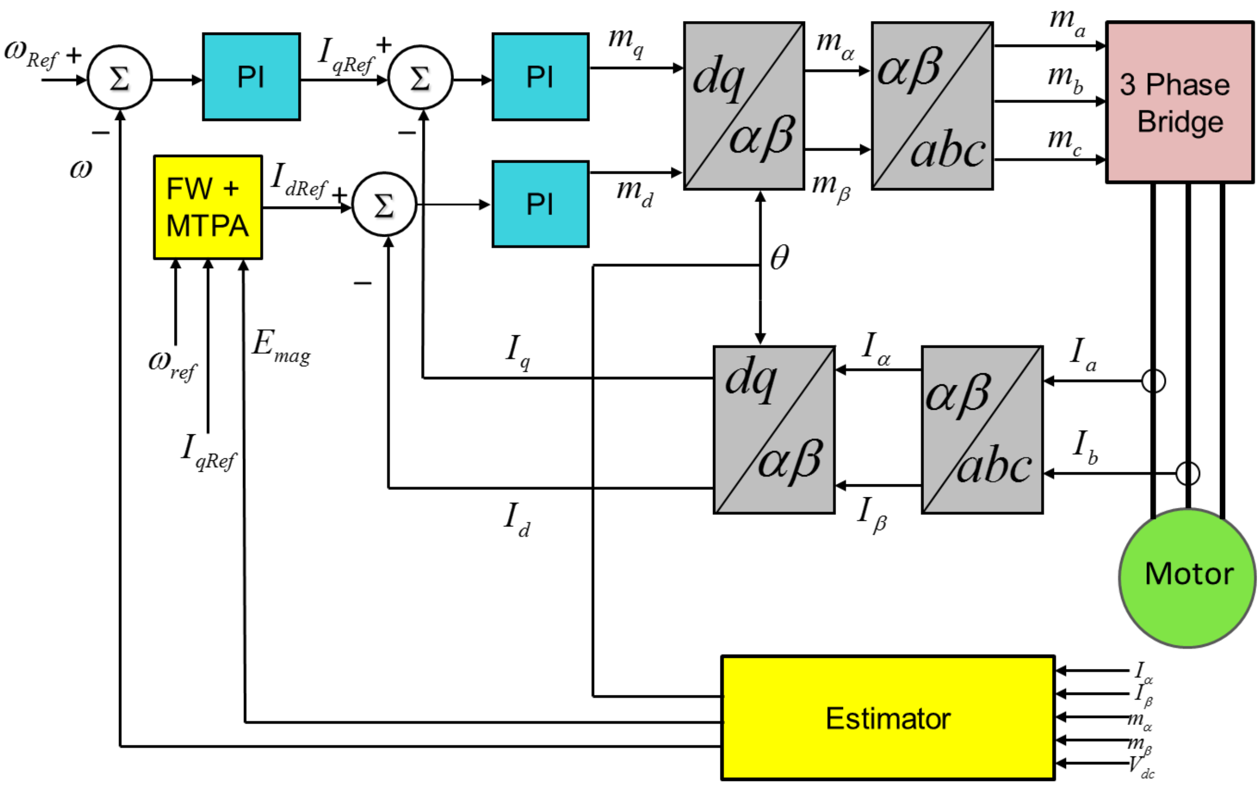


Figure 4.1: Field Oriented Control (FOC) Block Diagram

4.2 Control State Machine

The estimator gives reliable position information only beyond certain speeds. Hence, it is needed to start the motor in open speed loop and closed current loop. The entire starting sequence and $md*$ and $mq*$ generation is handled in control state machine.

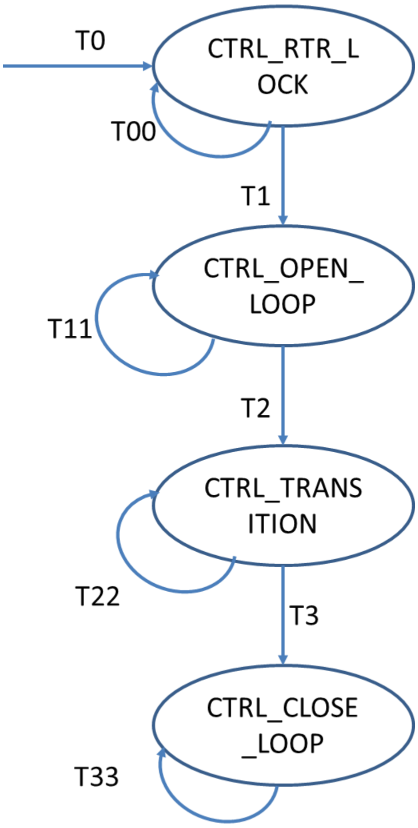


Figure 4.2: Control State Machine

4.2.1 State Description

Control state description is as follows:

S.No.	State	Description
1	CTRL_RTR_LOCK	Rotor is being locked to predefined angle w.r.t. stator
2	CTRL_OPEN_LOOP	Motor is running in speed open loop, but current closed loop
3	CTRL_TRANSITION	Current is decreased for better transitioning from open loop to closed loop
4	CTRL_CLOSE_LOOP	Motor is running in complete closed loop - FOC

4.2.2 Inner Current Loops

Since current loop is common to all the states, the current loop controller is executed outside the state machine.

$$m_d = PI(I_{d\mathrm{Re}f} - I_d)$$
$$PIQOutMax = \sqrt{VMAX\_SQ - m_d^2}$$
$$m_q = PI(I_{q\mathrm{Re}f} - I_q)$$

4.3 Motor Start-up

The rotor angle is not known initially. Since the motor is expected to run in sensor-less mode, it needs to be running at sufficient speed for the estimator to extract the speed and angle information accurately. Hence, the following sequence is undertaken:

1. Lock the rotor to a known angle. Knowing the initial angle helps in starting the motor. This is executed in the state CTRL_RTR_LOCK
2. Start the motor in forced commutation till sufficient speed is reached. This is executed in the state CTRL_OPEN_LOOP. While the motor is running in open loop, estimate the minimum current needed for the motor to run in open loop. If the current is ramped down to this minimum value, the transition to closed loop is expected to be smooth
3. Once the minimum current is estimated, reduce the motor current to this minimum value. This is achieved in CTRL_TRANSITION state. The current is reduced in small steps only when the current error is within a specific tolerance. This ensures that the motor remains stable in this state, as sudden decrease of the current could lead to motor stall
4. Once the reference is achieved to be the minimum current needed for the motor to run in open loop, calculate the angle difference between forced commutation angle (at which the motor is running now) and the estimated angle (at which the motor is expected to run in closed loop). Enable the speed controller and forcefully decrease this angle difference only when speed error is within a specific tolerance. This ensures that the motor doesn't speed up suddenly during the transition and ensures that it is in stable operation. Once the error is reduced to zero, the motor is running in FOC

4.4 Estimator – Angle Tracking PLL (ATPLL)

The estimator is back EMF based estimator which is based on motor stator voltage equations. Block diagram is as follows:

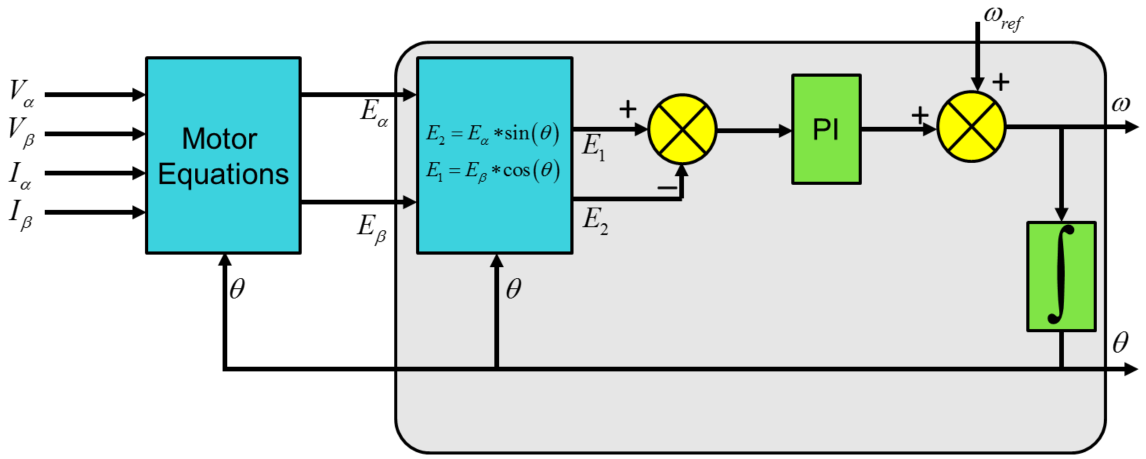


Figure 4.3: ATPLL Block Diagram

4.4.1 Motor Equations

Following are the motor equations in stator reference frame:

$$\begin{bmatrix} \Psi_{ds} \\ \Psi_{qs} \end{bmatrix} = \begin{bmatrix} L_{ds} & 0 \\ 0 & L_{qs} \end{bmatrix} \begin{bmatrix} I_{ds} \\ I_{qs} \end{bmatrix} + \begin{bmatrix} \Psi_{PM} \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} V_\alpha \\ V_\beta \end{bmatrix} = \begin{bmatrix} r_s & 0 \\ 0 & r_s \end{bmatrix} \begin{bmatrix} I_\alpha \\ I_\beta \end{bmatrix} + \frac{d}{dt}\left(\begin{bmatrix} L_0 - L_1\cos(2\theta) & -L_1\sin(2\theta) \\ -L_1\sin(2\theta) & L_0 + L_1\cos(2\theta) \end{bmatrix} \begin{bmatrix} I_\alpha \\ I_\beta \end{bmatrix} \right) + \begin{bmatrix} E_\alpha \\ E_\beta \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} E_\alpha \\ E_\beta \end{bmatrix} = \begin{bmatrix} V_\alpha \\ V_\beta \end{bmatrix} - \begin{bmatrix} r_s & 0 \\ 0 & r_s \end{bmatrix} \begin{bmatrix} I_\alpha \\ I_\beta \end{bmatrix} - \frac{d}{dt}\left(\begin{bmatrix} L_0 - L_1\cos(2\theta) & -L_1\sin(2\theta) \\ -L_1\sin(2\theta) & L_0 + L_1\cos(2\theta) \end{bmatrix} \begin{bmatrix} I_\alpha \\ I_\beta \end{bmatrix} \right)$$

$$L_0 = \frac{L_{ds} + L_{qs}}{2}; L_1 = \frac{L_{ds} - L_{qs}}{2}$$

If the motor does not have saliency (like a surface mount PMSM), $L_1 = 0$.

4.4.2 Angle Tracking

Once the BEMFs along two axes are estimated, angle tracking is used to align the estimator angle with BEMF. Following section deals with plant derivation for estimator PI design.

4.4.2.1 *Plant Derivation*

Let θ_0 denote the rotor flux angle and θ denote the estimator angle

$$
\begin{aligned}
&E_\alpha = E\cos(\theta_0)\\
&E_\beta = E\sin(\theta_0)\\
&E_1 = E_\beta\cos(\theta)\\
&E_2 = E_\alpha\sin(\theta)\\
&\Rightarrow \Delta E = E_1 - E_2\\
&= E_\beta\cos(\theta) - E_\alpha\sin(\theta)\\
&= E\sin(\theta_0)\cos(\theta) - E\cos(\theta_0)\sin(\theta)\\
&= E\sin(\theta_0 - \theta)\\
&= \Psi_{PM}\,\omega^*\sin(\theta_0 - \theta)
\end{aligned}
$$

For small angle deviations the plant comes out to be

$$\Delta E = \Psi_{PM}\,\omega^*(\theta_0 - \theta)$$

This approximation is valid only around the speed at which the motor is operating. Hence, to linearize the system, a feed forward term of speed reference is added to the plant. Following is the block diagram for linearized estimator.

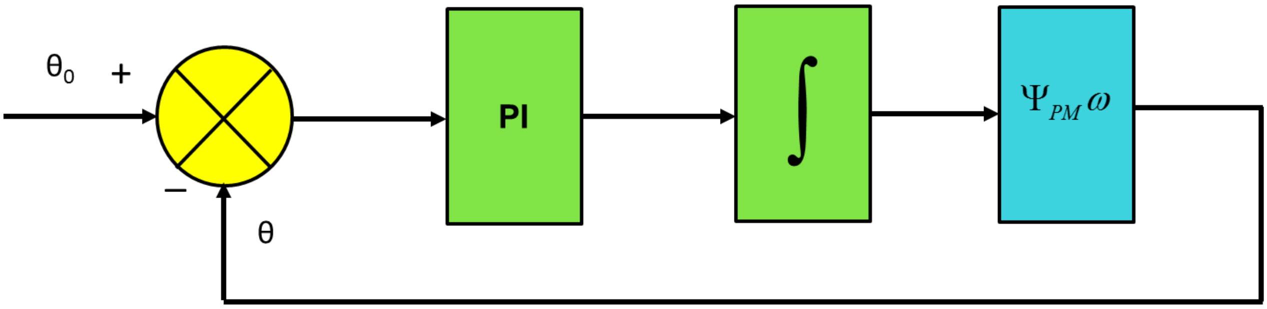


Figure 4.4: ATPLL Linearized Block Diagram

As can be seen, the plant is variable with speed. Hence, to compensate for this variation, the Ki is varied with speed till nominal speed and made equal to the speed reference. Beyond nominal speed, the Ki is held constant at nominal speed. This ensures sufficient Ki at high speeds, while compensating the effects of speed changes at low speeds.

*Note: Rotor angle is to be tracked. Since rotor angle is a ramp at constant speed, the open-loop transfer function of PLANT * CONTROLLER for estimator should have a double pole at origin. A PI has a pole at origin (K_i term). By treating the output of PI as speed and converting it to angle by integrating it, another pole at origin is placed (speed to angle integration), thus enabling to track angle without having a steady state error.*

4.4.3 AT-PLL Estimator Implementation Steps

Following are the detailed implementation steps for this estimator

1. Calculate BEMF based on motor voltage equations in section 4.4.1
2. Calculate BEMF error from angle tracking equations in section 4.4.2.1

3. Pass the error through a PI with saturation, Ki of which varies in accordance with speed as described in section 4.4.2
4. The output of PI is speed perturbation needed to align the estimator reference frame with BEMF. Filter this PI output
5. Add feed forward term of speed reference that has linearized the control system
6. The output is estimated speed, integral of which gives estimator reference frame orientation
7. Filtered estimated speed in step 6 is passed to speed controller as feedback

4.5 D-axis Current Reference Generation

This section deals with the D-axis current reference generation. The operation may be split into two parts:

1. Speed below nominal speed (for which the dc bus available is sufficient)
2. Speed above nominal speed (for which the dc bus available is insufficient)

4.5.1 Maximum Torque Per Ampere (MTPA)

This terminology deals with the maximizing the efficiency of the motor. This can be done by setting the d-axis current reference to zero for SPMSM, or by using reluctance torque to aid the electro-magnetic torque in case of IPMSM. The reluctance torque appears due to magnetic saliency of the motor, and hence, the following algorithm is applicable only for IPMSM. If the rotor flux is weakened by a very small amount, the motor produces a reluctance torque aiding the electromagnetic torque.

4.5.1.1 MTPA Current Reference

Since the voltage magnitude does not change with load, the motor current may not be in phase with BEMF. Hence, an active mechanism is needed to make current in phase with BEMF. This is done by changing the voltage reference.

MTPA algorithm takes this idea further and manages the voltage reference such that the motor operates at MTPA point.

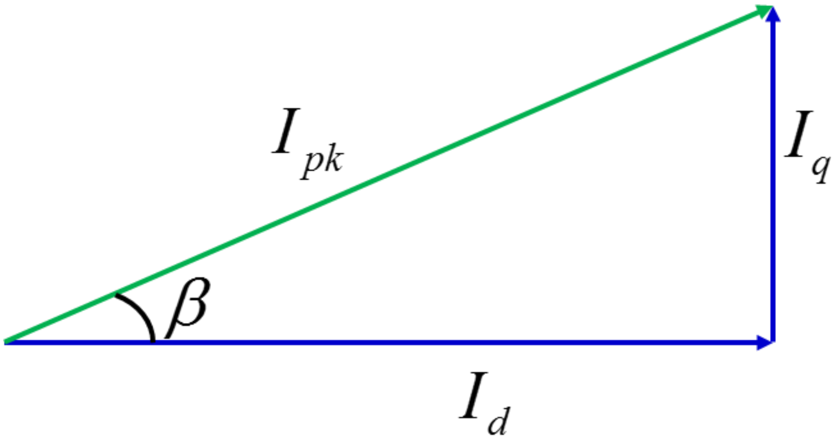


Figure 4.5: Relationship Between Currents

$$I_d = I_{pk}\cos(\beta)$$
$$I_q = I_{pk}\sin(\beta)$$
$$T_e = \frac{3}{2}\frac{P}{2}\left(\Psi_{PM} I_q + (L_{ds}-L_{qs}) I_d I_q\right)$$
$$\Rightarrow T_e = \frac{3}{2}\frac{P}{2} I_{pk}\left(\Psi_{PM}\sin(\beta) + (L_{ds}-L_{qs}) I_{pk}\cos(\beta)\sin(\beta)\right)$$

For maximum torque for the same current:

$$\frac{\partial T_e}{\partial \beta} = 0$$
$$\Rightarrow \frac{3}{2}\frac{P}{2} I_{pk}\left(\Psi_{PM}\cos(\beta) + (L_{ds}-L_{qs}) I_{pk}\left(\cos^2(\beta)-\sin^2(\beta)\right)\right) = 0$$
$$\Rightarrow \Psi_{PM} I_d + (L_{ds}-L_{qs})\left(I_d^2 - I_q^2\right) = 0$$
$$\Rightarrow (L_{ds}-L_{qs}) I_d^2 + \Psi_{PM} I_d - (L_{ds}-L_{qs}) I_q^2 = 0$$
$$\Rightarrow I_d = \frac{-\Psi_{PM} \pm \sqrt{\Psi_{PM}^2 + 4(L_{ds}-L_{qs})^2 I_q^2}}{2(L_{ds}-L_{qs})}$$
$$L_1 = \frac{L_{ds}-L_{qs}}{2}$$
$$\Rightarrow I_d = \frac{-\Psi_{PM} \pm \sqrt{\Psi_{PM}^2 + (4L_1 I_q)^2}}{4L_1}$$

For implementation purposes, this is implemented as follows:

$$I_d = \frac{-\omega\Psi_{PM} + \sqrt{\omega^2\Psi_{PM}^2 + (4\omega L_1 I_q)^2}}{4\omega L_1}$$

Assuming current and speed loops are fast enough, references are used instead of actual values. However, for BEMF, the actual value is used to avoid any impact of inaccuracy of BEMF constant.

$$I_{dRef\_MTPA} = \left|\begin{array}{l} \qquad 0 \text{ for SPMSM} \\ \\ \dfrac{-E_{mag} + \sqrt{E_{mag}^2 + (4\omega_{Ref} L_1 I_{qRef})^2}}{4\omega_{Ref} L_1} \text{ for IPMSM} \end{array}\right.$$

4.5.1.2 MTPA Current Reference Implementation Steps

Following are the detailed implementation steps for equation based MTPA current referencing

1. Calculate $\omega_{Ref} L_1$
2. Obtain E_{mag} from the estimator
3. Considering appropriate scaling for I_{qRef} calculate I_{dRef_MTPA} as detailed in Section 4.5.1.1

D – Axis current reference is generated such that at the minimum MTPA current reference is applied to the motor in case of IPMSM, whereas only negative current is applied in case of PMSM flux weakening region.

4.5.2 Flux Weakening

The BEMF increases linearly with motor speed increase. Hence, to counter this and support load, the applied voltage increases linearly with speed. Beyond a certain speed, the applied voltage can no longer increase as dc link is not sufficient. Other practical considerations like winding insulation, motor safe operating zone, etc. are considered to set the dc link voltage to acceptable levels.

This speed is nominal speed for a given dc link voltage and motor. Beyond this speed, airgap flux is decreased to increase the speed. This compromises on torque capability of the motor to gain on speed range.

4.5.2.1 Flux Weakening Current Reference – Equation Based

Motor steady state equations are used for estimating current reference needed to maintain applied voltage on the voltage limit circle. It is achieved in open loop by estimating V_{qs} needed for a given V_{ds} and is calculated as follows:

$$V_{ds\_fw} = r_s I_{dRef} - \omega L_{qs} I_q$$
$$V_{qs\_fw} = r_s I_q + \omega L_{ds} I_d + \omega \Psi_{PM}$$
$$\sqrt{V_{ds\_fw}^2 + V_{qs\_fw}^2} = V_{\max}$$
$$\Rightarrow V_{qs\_fw} = \sqrt{V_{\max}^2 - V_{ds\_fw}^2} = r_s I_q + \omega L_{ds} I_d + \omega \Psi_{PM}$$
$$\Rightarrow I_{dRef\_fluxWeak} = \frac{V_{qs\_fw} - r_s I_q - \omega \Psi_{PM}}{\omega L_{ds}}$$

These equations signify the default equation based flux weakening scheme. In the modified version of this scheme, the BEMF used in the above equation $\left(\omega \Psi_{PM}\right)$ is calculated from estimator $\left(E_{mag}\right)$. This version can be selected by enabling “ACTUAL_BEMF_EQN_BASED” in fdweak.h

4.5.2.2 Flux Weakening Current Reference Implementation Steps – Equation Based

Following are the detailed implementation steps for equation based flux weakening current reference

1. Calculate V_{ds_fw} from previous I_{dRef} and current I_q
2. Calculate V_{qs_fw} needed to maintain voltage magnitude at $V_{\max}$

3. Calculate $I_{dRef\_fluxWeak}$ as per equation shown in section 4.5.2.1

4.5.2.3 Flux Weakening Current Reference – Closed Loop

The equation based flux weakening described in sections 4.5.2.1 and 4.5.2.2 ensures that the applied voltage magnitude is very close to the voltage circle limit, but does not ensure that it always lies on the voltage circle. If such a control is achieved, it results in the most efficient flux weakening. Closed loop flux weakening ensures that the applied voltage magnitude lies on the voltage limit circle. The following block diagram describes this control strategy:

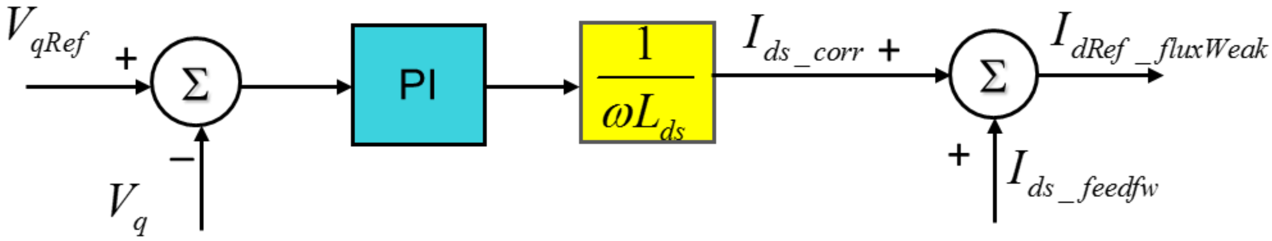


Figure 4.6: Closed Loop Flux Weakening Implementation Block Diagram

The following gives the calculation of V_{qRef} and I_{ds_feedfw} :

$$V_{qRef} = \sqrt{V_{\max}^2 - V_d^2}$$

$$I_{ds\_feedfw} = \frac{V_{qRef} - \left(r_s * I_{qRef} + L_{qs} * \frac{dI_{qRef}}{dt} + E_{mag} \right)}{\omega_{Ref} * L_{ds}}$$

Where E is the actual BEMF calculated from speed estimator.

By making the control loop structure as shown, the plant for PI becomes a unity gain, leading to any PI with reasonable bandwidth and saturation to work with the algorithm.

Only feed-forward path can also be chosen by enabling “CLOSED_LOOP_FEEDFWD” in fdweak.h

Closed loop path (adding the PI output) is chosen by enabling “CLOSED_LOOP_FW” in fdweak.h

4.5.3 Choice Between MTPA and Flux Weakening Current References

Since during flux weakening, we sacrifice the efficiency for extended speed operation, we choose the reference which is more negative.

Hence, $I_{dRef} = \min\left(I_{dRef\_MTPA} , I_{dRef\_fluxWeak} \right)$

5 Application Specific Algorithms

This section deals with implemented algorithms that are unique to each application. The sections 3 and 0 deal with a generic motor application and form the base code structure for running a permanent magnet synchronous motor. However, different applications need different algorithms to meet their specific needs.

5.1 Washing Machine Application

This section deals with application specific algorithms for washing machine application.

5.1.1 Requirement

This application requirement comes with a combination of high inertia of the machine and the possibility of the motor running in very high speed.

If the motor is stopped when it is running in deep flux weakening:

1. By removing the gate pulses to power switches: the energy of the motor is dumped into the dc bus, increasing it to possibly unsafe levels
2. By braking the motor by shorting the motor terminals through the lower power switches: large current flows through the power switches due to large BEMF of the motor at that instance.

5.1.2 Application Specific Algorithm – Soft Stop

To overcome this, whenever the motor is commanded stop, the motor speed is ramped down in a controlled manner and then stopped. This would require a modification in the main state machine of Figure 3.1 and is highlighted below:

5.1.2.1 Main State Machine Modification

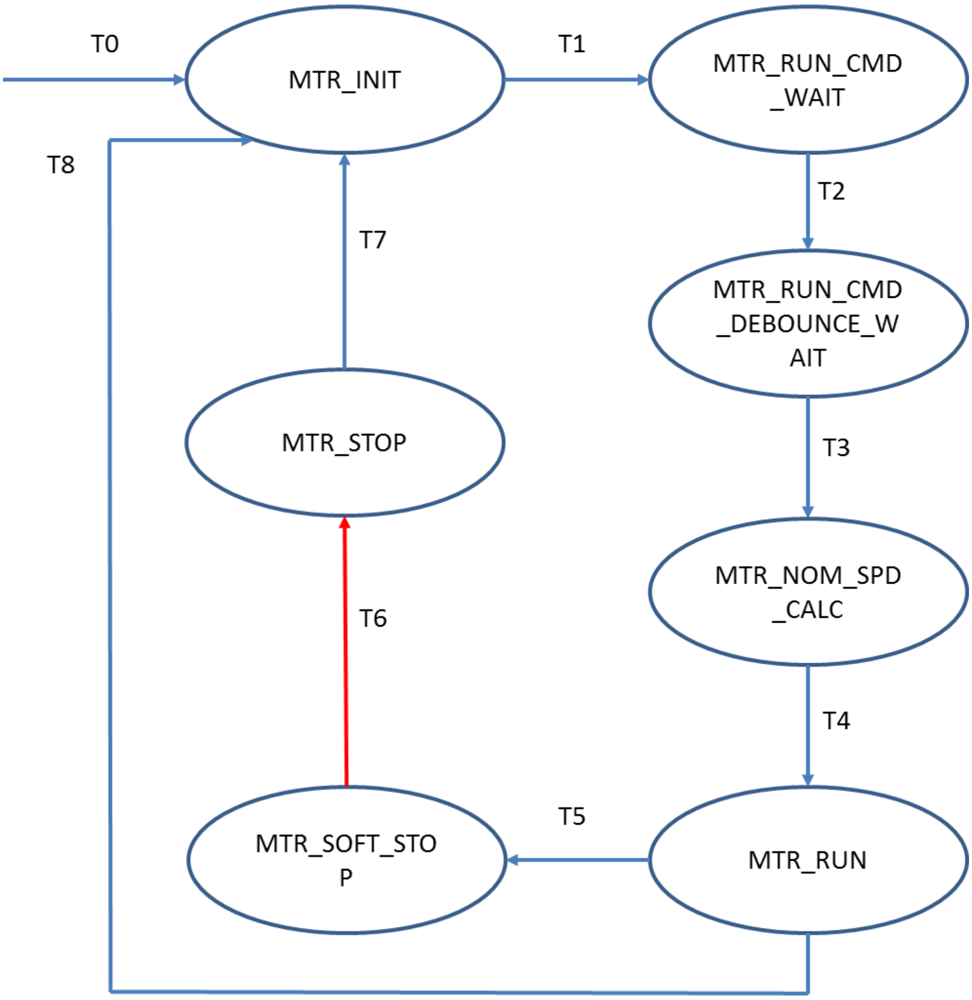


Figure 5.1: Main State Machine Modification for Washing Machine: Soft Stop

5.2 Compressor Application

This section deals with application specific algorithms for compressor application.

5.2.1 Requirement

This application requirement comes from the nature of the load. A compressor will be loading the motor in a cyclic manner. This would produce vibrations in the motor and compressor, leading to vibrations in the pipes that carry the gas. Due to the vibrations, the pipes get fatigued and their life may come down.

Hence, the requirement is that these vibrations should be as less as possible.

5.2.2 Application Specific Algorithm – Torque Compensation

To overcome this, the vibration is controlled by controlling the motor. This would require a modification in the basic FOC block diagram of Figure 4.1.

5.2.2.1 FOC Block Diagram Modification

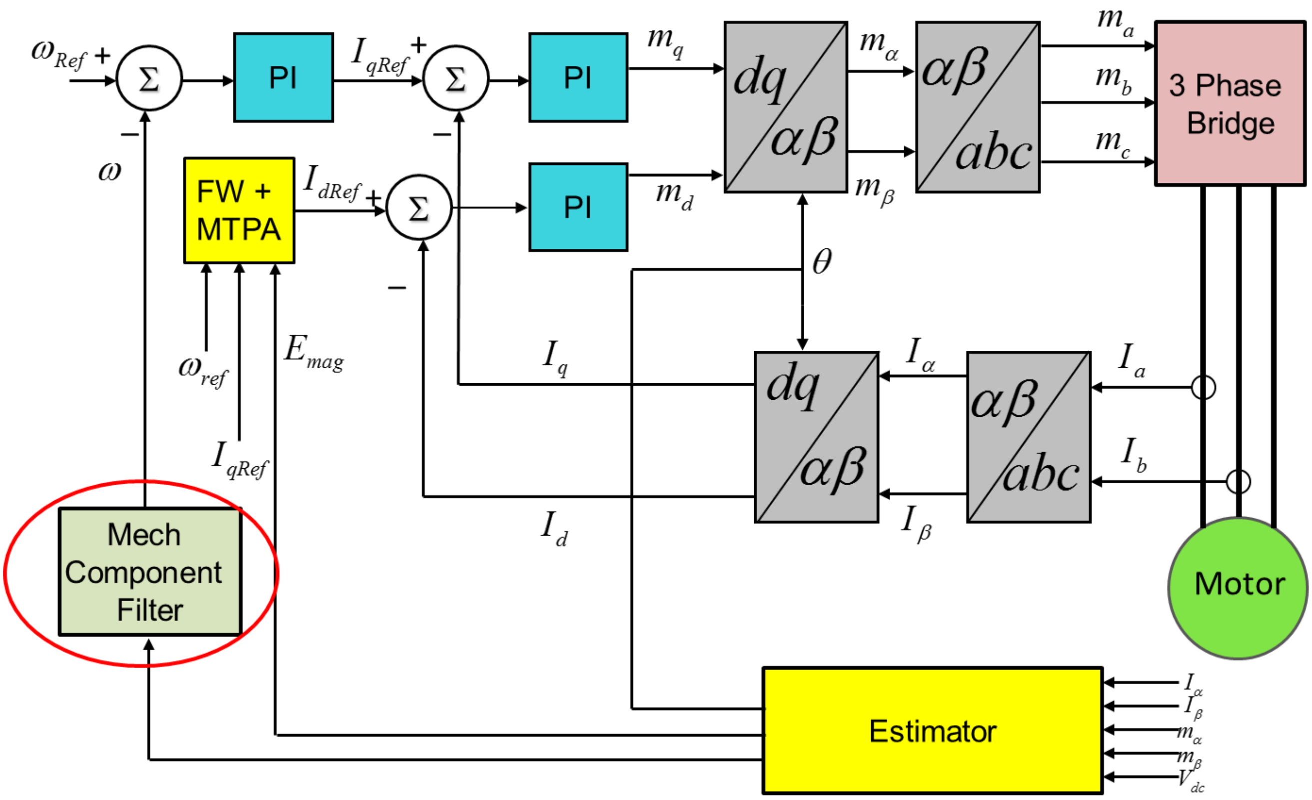


Figure 5.2: Field Oriented Control Block Diagram Modification for Compressor: Torque Compensation

5.3 Fan Application

This section deals with application specific algorithms for fan application.

5.3.1 Requirement

Fan application has a set of requirements that are unique due to legacy. The older fans being used were single phase induction motor fans, where the fans used to start from whatever position they are in and could catch the speed on the fly in case of a power reset. Since the motor now being used is a PMSM, these legacy features are expected to be incorporated in these applications as well.

The requirements can be summed up as follows:

1. Start the fan from its present standstill position. There should be no discernible backwards movement of the blades
2. If the fan is running in forward direction, it should be capable to catch the speed on the fly in case of a power reset
3. If the fan is running in the reverse direction, it should be stopped from doing so before running in the forward direction

5.3.2 Application Specific Algorithms – Initial Position Detection (IPD) and Wind-milling

To address the requirements discussed in Section 5.3.1, there are modifications needed in the main state machine of Figure 3.1, along with state machines for each of the algorithm.

5.3.2.1 Main State Machine Modification

The following is the modification for main state machine specific to fan application.

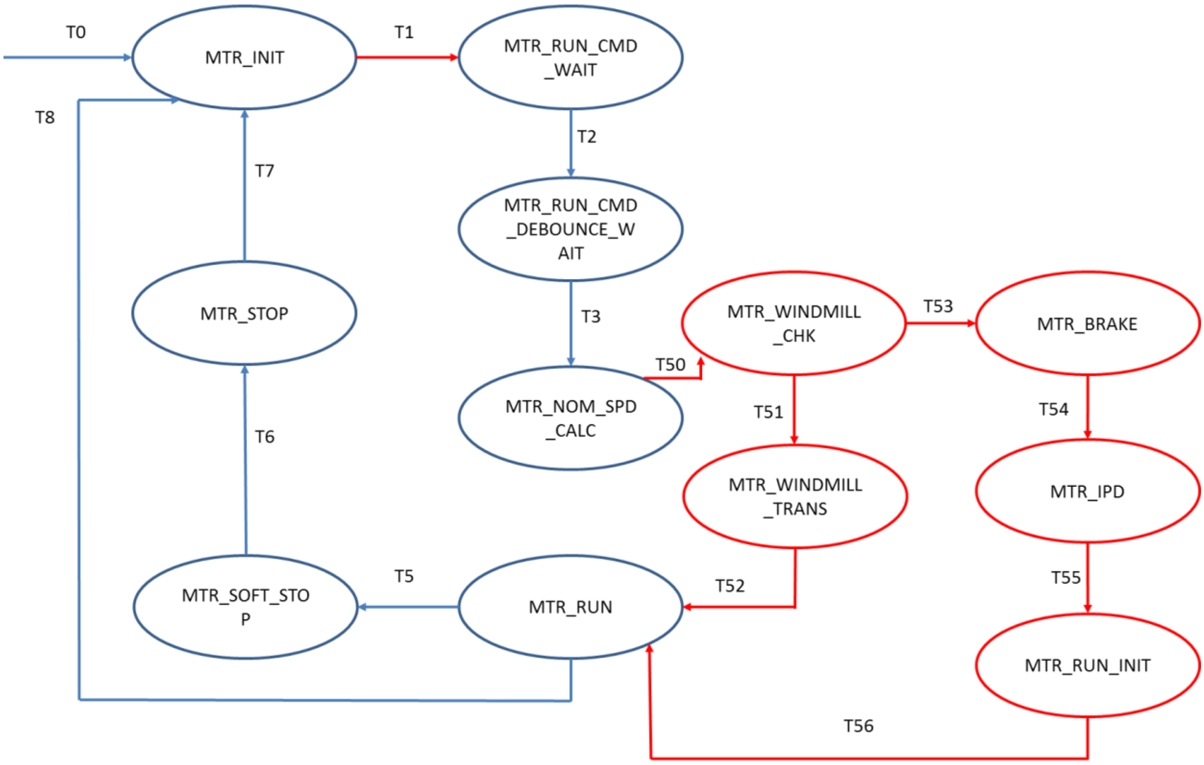


Figure 5.3: Main State Machine Modification for Fan Application – IPD and Wind-milling

5.3.2.1.1 State Description

Newly added state description is as below:

S.No.	State	Description
1	MTR_WINDMILL_CHK	Checking if the motor is wind-milling, and if it is, the direction and speed of the motor
2	MTR_WINDMILL_TRANS	Transition state from wind-milling to closed loop where integrators are initialized
3	MTR_BRAKE	Braking of the motor
4	MTR_IPD	Initial Position Detection state
5	MTR_RUN_ INIT	Initialize variables prior to running of the motor

5.3.2.2 Wind-milling

For wind-milling detection, the BEMFs of the motor are to be sensed. One phase BEMF is sufficient to detect if the motor is wind-milling or not, however, at least two phase BEMFs are needed to detect the direction of rotation as well.

5.3.2.2.1 BEMF Sampling

In this implementation, phase A and phase B BEMFs are sensed sequentially. Hence, the shared ADC core (ADC0) is used for this purpose. The BEMFs E_a and E_b are sensed every alternate PWM cycle with the wind-milling state machine being executed only at the end of E_b sampling.

5.3.2.2.2 Wind-milling State Machine

The following is the wind-milling state machine:

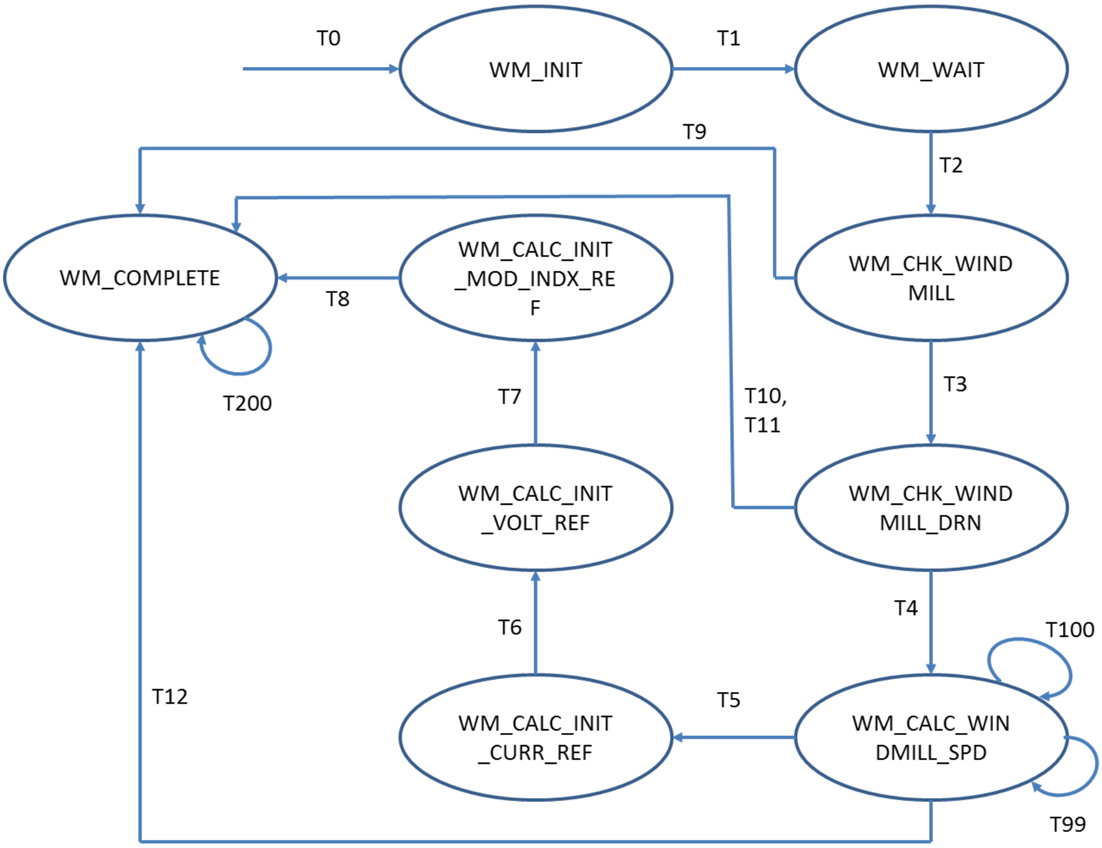


Figure 5.4: Wind-milling State Machine

5.3.2.2.2.1 State Description

The following is the state description for wind-milling state machine:

S.No.	State	Description
1	WM_INIT	Initialize Windmilling variables
2	WM_WAIT	Wait for start command for windmilling
3	WM_CHK_WINDMILL	Check if the motor is windmilling
4	WM_CHK_WINDMILL_DRN	Check the motor direction during windmilling
5	WM_CALC_WINDMILL_SPD	Calculate the windmilling speed
6	WM_CALC_INIT_CURR_REF	Calculate initialization current references
7	WM_CALC_INIT_VOLT_REF	Calculate initialization voltage references
8	WM_CALC_INIT_MOD_INDX_REF	Calculate initialization modulation index references
9	WM_COMPLETE	Windmilling is complete

5.3.2.3 Initial Position Detection (IPD)

Initial Position Detection (IPD) relies on saliency of the motor when excited at high enough frequency. Saliency indicates that inductance is different at each position. By exciting each possible combination and analysing the current response, the rotor position can be deduced.

The basis of IPD is as follows:

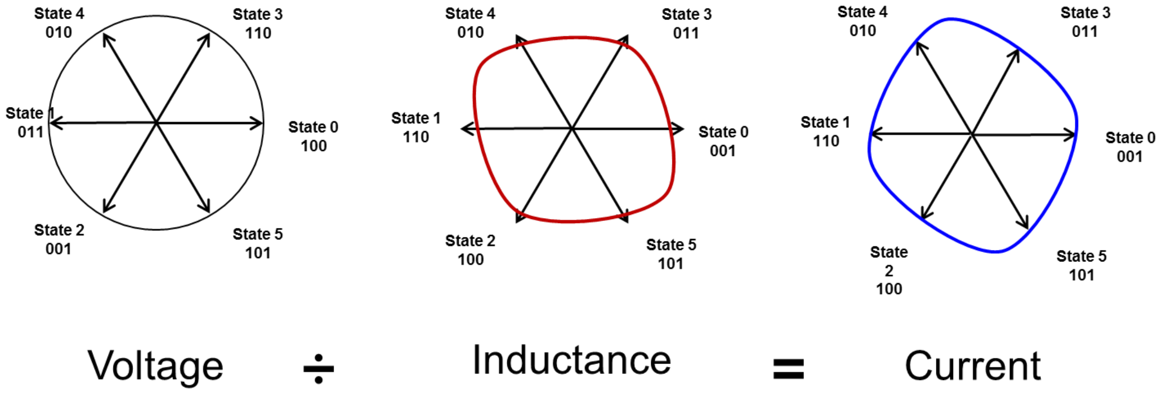


Figure 5.5: Impact of Inductance Variation on Current – IPD

By exciting each state for a time that is less than the motor time constant, the current magnitude variation will give the rotor position.

5.3.2.3.1 Bus Current Sampling

IPD requires bus current to be sampled. In this implementation, the shared core (ADC0) is used to sample the bus current.

5.3.2.3.2 PWM Array Sequencing

Taking the state sequencing of Figure 5.5 as a-b-c:

a	b	c
1	0	0
1	1	0
0	1	0
0	1	1
0	0	1
1	0	1

It can be observed that each phase is having a bottom ON (0) and a top ON (1) exactly 3 times. Considering ‘c’ as reference, the sequence of ‘b’ is same as that of ‘c’, but offset by 2 counts. Similarly, the sequence of ‘a’ is same as that of ‘c’, but offset by 4 counts. Hence, only one array is needed, and a, b, c sequencing can be achieved by suitably offsetting the indices.

5.3.2.3.3 IPD State Machine

Following is the state machine for IPD:

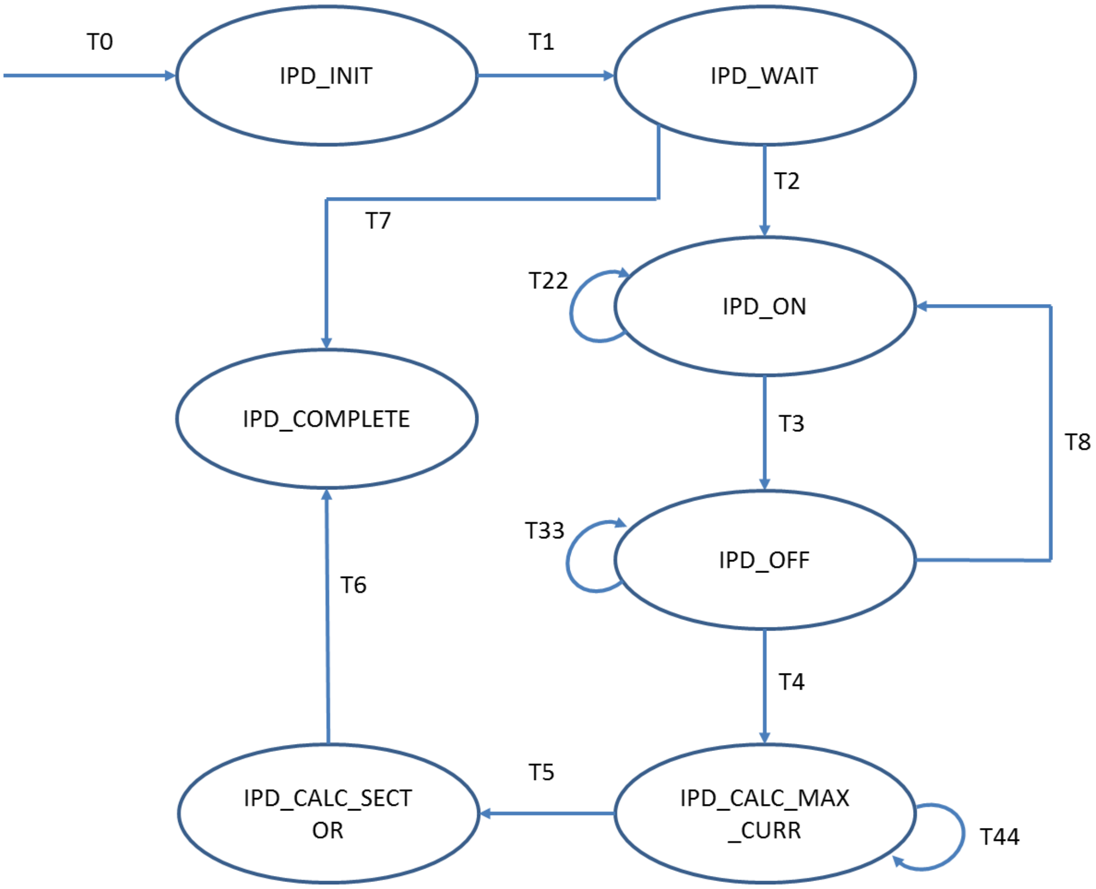


Figure 5.6: IPD State Machine

5.3.2.3.3.1 State Description

The following is the state description for IPD state machine:

S.No.	State	Description
1	IPD_INIT	Initialize IPD variables
2	IPD_WAIT	Wait for command for IPD
3	IPD_ON	Turn on the pulses for IPD
4	IPD_OFF	Turn off the pulses for IPD
5	IPD_CALC_MAX_CURR	Calculate the maximum current
6	IPD_CALC_SECTOR	Calculate the sector where the rotor is
7	IPD_COMPLETE	IPD is complete

6 Test Results

This section deals with tests performed on washing machine (SPMSM), compressor (IPMSM) and fan (SPMSM) and their results.

6.1 Washing Machine

This section deals with washing machine test results. The washing machine application is taken as an example of Surface mount Permanent Magnet Synchronous Motor (SPMSM).

6.1.1 Test Setup

Tests were performed on following setup:

1. Washing machine having model # DG7528BS
2. Chroma 61504 as the power source.
3. MCHV-2 Board
4. External Op-AMP PIM of dsPIC33EP256MC506
5. MPLABX Ver 4.05; XC16 Ver 1.33

6.1.2 Test Conditions

The washing machine was loaded with:

S.No.	Item	Qty	Remarks
1	Formal Trousers	3	
2	Fleece Trousers	1	Soaks Lot of water
3	Denim Pant	1	Heavy
4	Denim Pouch	1	
5	Formal Shirts	1	
6	T-Shirts	3	
7	Fleece Windcheater	1	Soaks Lot of water

The input voltage to MCHV2 was maintained at 220V ac.

6.1.3 Test Results

The following are the test results under various modes of operation of the washing machine.

6.1.3.1 Washing Cycle

The clothes were soaked in water prior to washing cycle. The motor was running at 50 rpm continuously for 30 minutes and the data was recorded every 5 minutes.

Following is the recorded data summary for washing cycle

Time (min)	Source Volt (V)	Source Curr (A)	Source Pwr (W)
0	220	0.49	48
5	220	0.43	40.4
10	220	0.49	48.7
15	220	0.46	50.5
20	220	0.48	55.2
25	220	0.47	51.6
30	220	0.49	60.1

Please note that the power and current depends upon load distribution. The indicated values are instantaneous.

6.1.3.2 Spinning Cycle

The drain valve was kept open and the motor was spun at 1000 rpm for 12 minutes. Data was recorded every 2 minutes.

Following is the recorded data summary for washing cycle

Time (min)	Source Volt (V)	Source Curr (A)	Source Pwr (W)
0	220	2.8	392
2	220	2.53	334
4	220	2.35	318
6	220	2.47	311
8	220	2.46	313
10	220	2.5	324
12	220	2.52	309

As the water gets drained out of clothes, the power drawn from source keeps decreasing.

6.2 Air Conditioner Compressor

This section deals with Air Conditioner test results. Air conditioner outdoor unit compressor application is taken as an example of Interior Permanent Magnet Synchronous Motor (IPMSM).

6.2.1 Test Setup

Tests were performed on following setup:

1. Compressor Model # ASM108D18UFZA
2. Chroma 61504 as the power source.
3. MCHV-2 Board
4. External Op-AMP PIM of dsPIC33EP256MC506
5. MPLABX Ver 4.05; XC16 Ver 1.33

6.2.2 Test Conditions

Input voltage to MCHV2 maintained at 220V ac.

6.2.3 Test Results

The following are the test results of Air Conditioner Compressor under different speeds of operation.

Speed (rpm)	Source Volt (V)	Source Curr (A)	Source Pwr (W)
500	220	0.41	42
1000	220	0.92	106
1500	220	1.34	186
2000	220	2.14	280
2500	220	2.77	372
3150	220	3.53	484

6.3 Refrigerator Compressor

This section deals with Refrigerator compressor test results. Refrigerator compressor application is taken as an example of Interior Permanent Magnet Synchronous Motor (IPMSM).

6.3.1 Test Setup

Tests were performed on following setup:

1. Compressor Model # BSA057NHMV
2. Chroma 61504 as the power source.
3. MCHV-2 Board
4. External Op-AMP PIM of dsPIC33EP256MC506
5. MPLABX Ver 4.05; XC16 Ver 1.33

6.3.2 Test Conditions

Input voltage to MCHV2 maintained at 220V ac.

6.3.3 Test Results

The following are the test results of Refrigerator Compressor under different speeds of operation.

Speed (rpm)	Source Volt (V)	Source Curr (A)	Source Pwr (W)
1500	220	0.41	46
2500	220	0.49	60
3000	220	0.55	67
4220	220	0.6	72

6.4 High Voltage Fan

This section deals with high voltage fan test results. Fan application is taken as an example of Surface mount Permanent Magnet Synchronous Motor (SPMSM).

6.4.1 Test Setup

Tests were performed on following setup:

1. AC Outdoor unit Fan
2. Chroma 61502 as the power source.
3. MCHV-2 Board modified as per following:
   3.1 Jumpers J12, J13 on voltage, J14 on current
   3.2 Remove R104, R105 on the power board
   3.3 Short R122, R123 on the power board
4. External Op-AMP PIM of dsPIC33EP256MC506 with PFC_EXT_OP_AMP matrix board

MPLABX Ver 4.05; XC16 Ver 1.33

6.4.2 Test Conditions

Input voltage to MCHV2 maintained at 220V ac.

6.4.3 Test Results

The following are the test results of high voltage fan under different speeds of operation.

Speed (rpm)	Source Volt (V)	Source Curr (A)	Source Pwr (W)
200	220	0.14	15.6
300	220	0.16	18.1
400	220	0.2	22.9
500	220	0.25	30.5
600	220	0.34	41.7
700	220	0.48	58.7
800	220	0.65	81.1
900	220	0.87	108.9
1000	220	1.14	147.9

6.5 Low Voltage Fan

This section deals with low voltage fan test results. Fan application is taken as an example of Surface mount Permanent Magnet Synchronous Motor (SPMSM).

6.5.1 Test Setup

Tests were performed on following setup:

1. Microchip low voltage fan
2. Scientific power supply as the power source.
3. MCLV-2 Board modified as per following:
   3.1 Jumpers JP1, JP2 on voltage, JP3 on current
4. External Op-AMP PIM of dsPIC33EP256MC506 with PFC_EXT_OP_AMP matrix board

MPLABX Ver 4.05; XC16 Ver 1.33

6.5.2 Test Conditions

Input voltage to MCLV2 maintained at 24V dc.

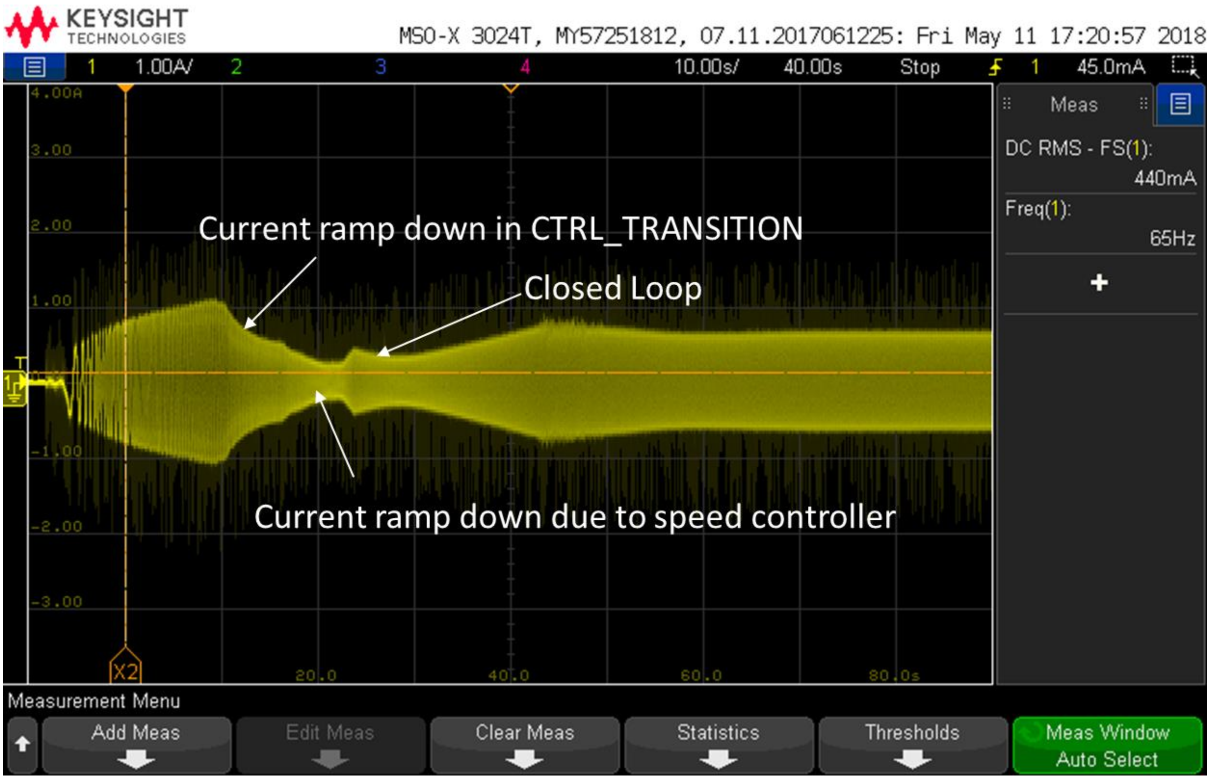
6.5.3 Test Results

The following are the test results of low voltage fan under different speeds of operation.

Speed (rpm)	Source Volt (V)	Source Curr (A)	Source Pwr (W)
100	24	0.16	3.84
150	24	0.21	5.04
200	24	0.29	6.96
250	24	0.4	9.6
320	24	0.62	14.88

6.6 Start-up

The following figure shows the effect of start-up sequencing on fan application, where the impact is more visible.


Current ramp down in CTRL_TRANSITION

Closed Loop

Current ramp down due to speed controller

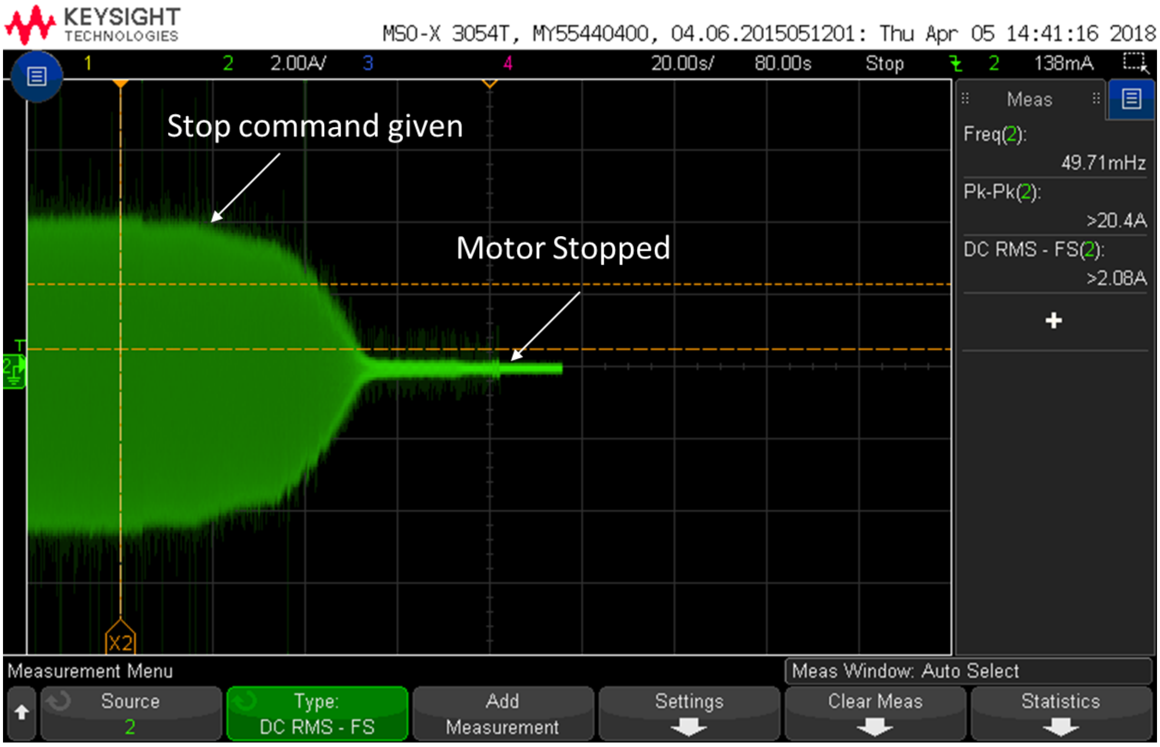
Figure 6.1: Start-up routine

6.7 Application Specific Algorithms

This section deals with waveforms and test results specific to each algorithm.

6.7.1 Washing Machine

The soft stop for washing machine implemented gives a current response as shown below:



MSO-X 3054T, MY55440400, 04.06.2015051201: Thu Apr 05 14:41:16 2018

1 2 2.00A/ 3 4 20.00s/ 80.00s Stop 2 138mA

Freq(2): 49.71mHz

Pk-Pk(2): >20.4A

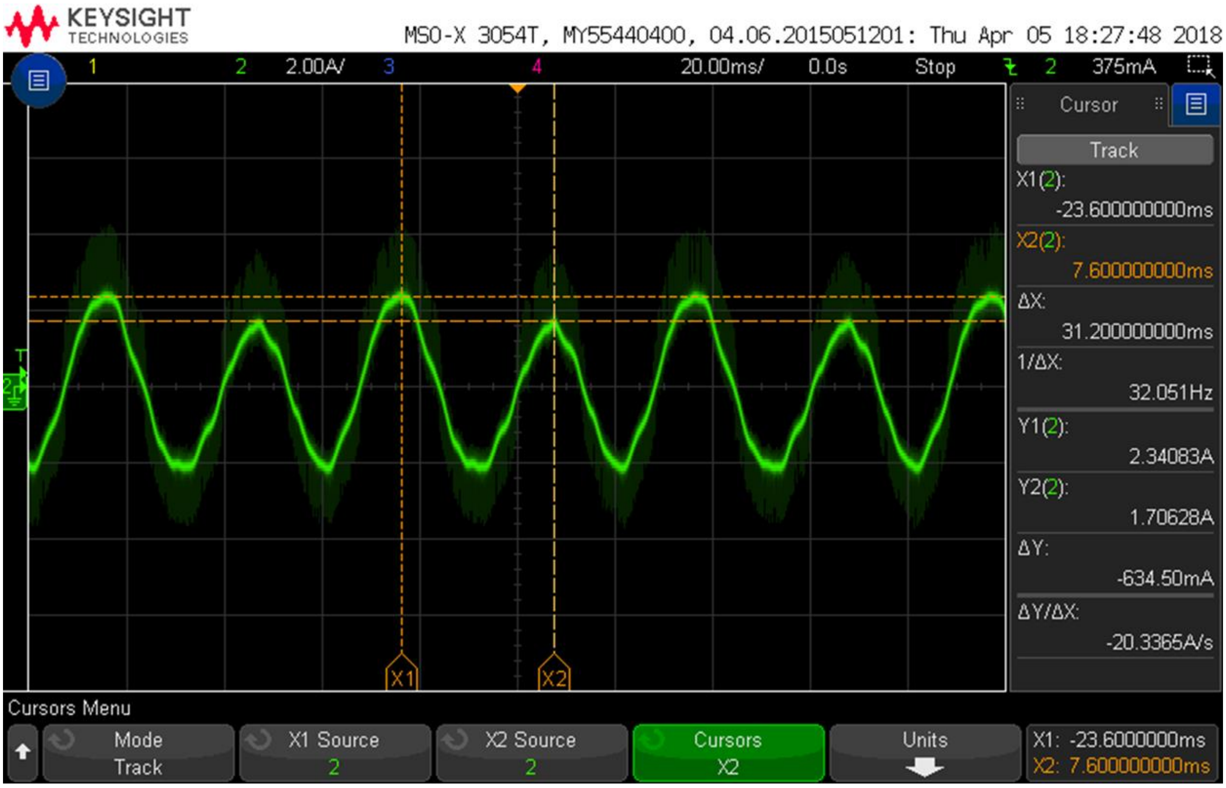
DC RMS - FS(2): >2.08A

Meas Window: Auto Select

Figure 6.2: Washing Machine - Soft Stop

6.7.2 Compressor

The torque compensation waveform for a compressor is as shown below:



MSO-X 3054T, MY55440400, 04.06.2015051201: Thu Apr 05 18:27:48 2018

1 2 2.00A/ 3 4 20.00ms/ 0.0s Stop 2 375mA

X1(2): -23.600000000ms

X2(2): 7.600000000ms

ΔX: 31.200000000ms

1/ΔX: 32.051Hz

Y1(2): 2.34083A

Y2(2): 1.70628A

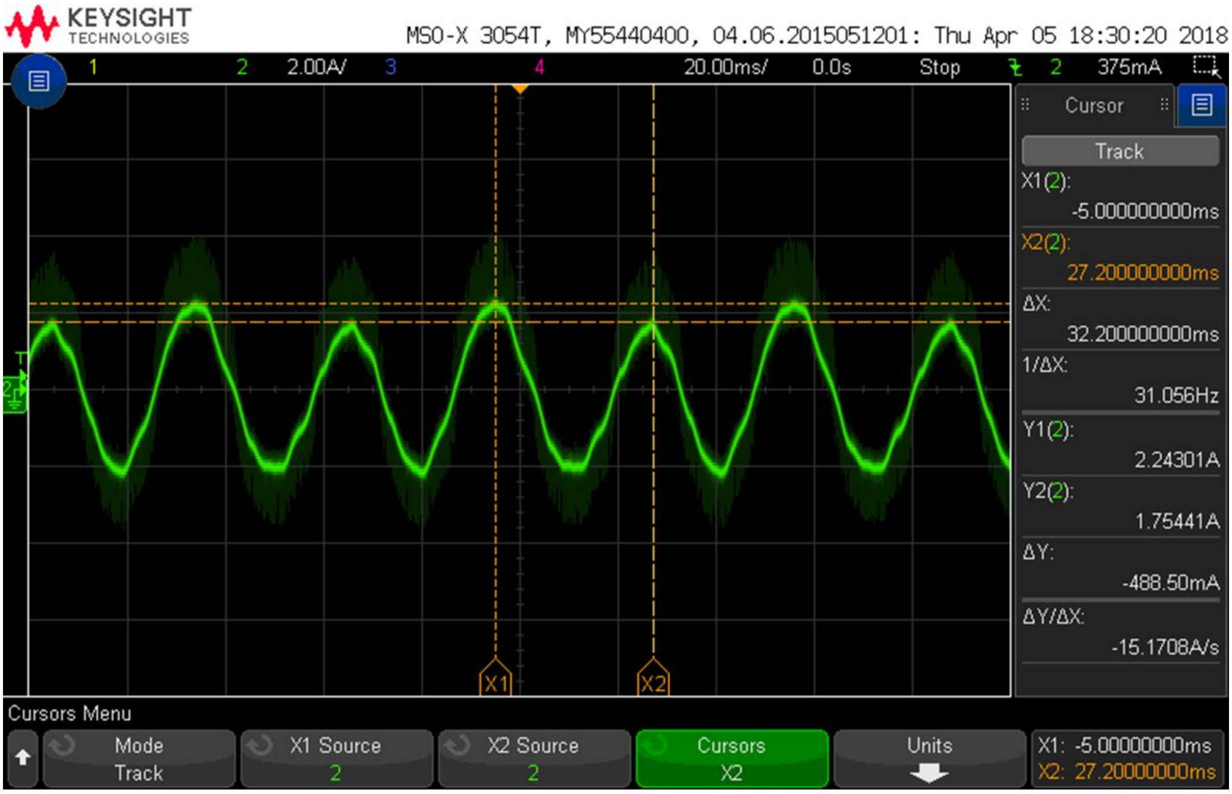
ΔY: -634.50mA

ΔY/ΔX: -20.3365A/s

X1: -23.6000000ms

X2: 7.600000000ms

Figure 6.3: Compressor: Current without Torque Compensation



MSO-X 3054T, MY55440400, 04.06.2015051201: Thu Apr 05 18:30:20 2018

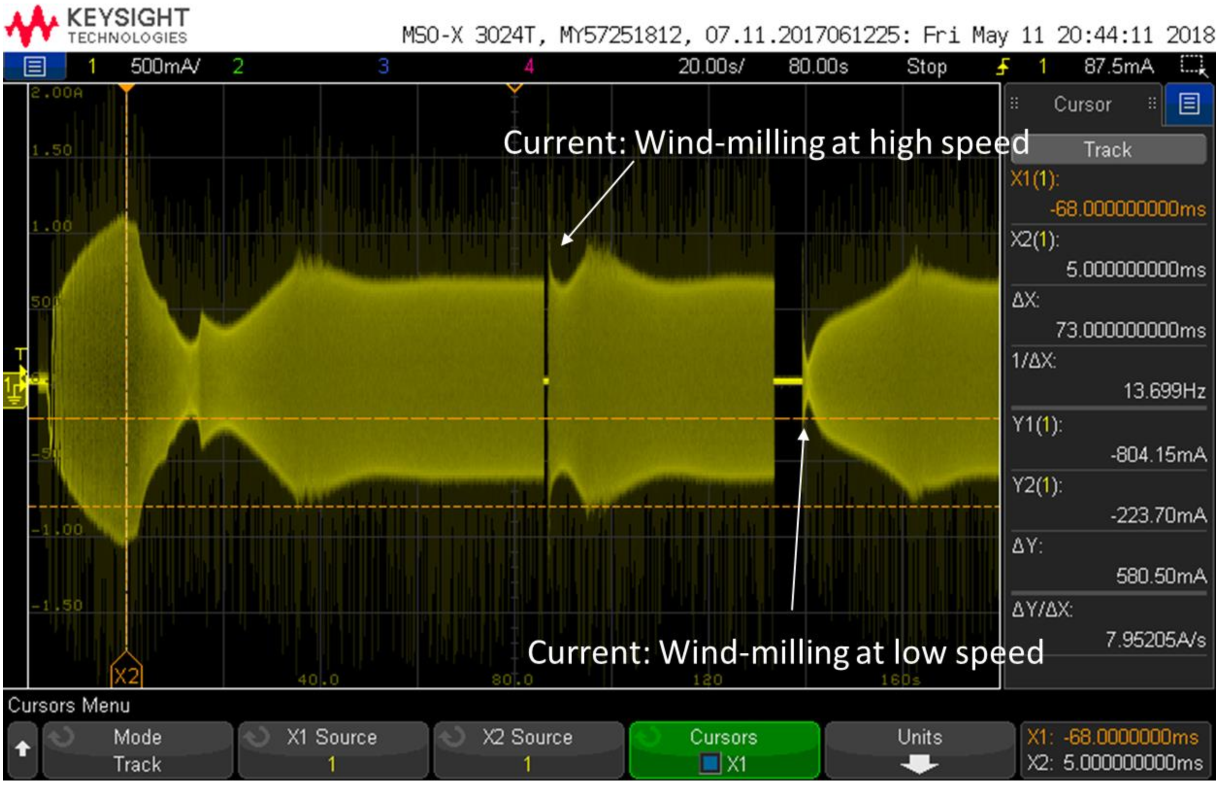
X1(2): -5.000000000ms
X2(2): 27.200000000ms
ΔX: 32.200000000ms
1/ΔX: 31.056Hz
Y1(2): 2.24301A
Y2(2): 1.75441A
ΔY: -488.50mA
ΔY/ΔX: -15.1708A/s

X1: -5.00000000ms
X2: 27.20000000ms

Figure 6.4: Compressor: Current with Torque Compensation

6.7.3 Fan

The waveforms for wind-milling for fan application are as shown below:



Current: Wind-milling at high speed

Current: Wind-milling at low speed

Figure 6.5: Fan: Wind-milling at different speeds

7 Conclusion

Following conclusions can be drawn based on the test results:

1. This document has presented a unified approach for IPMSM and SPMSM motors
2. The estimator presented (ATPLL) can be used seamlessly for SPMSM and IPMSM motors
3. The flux weakening algorithm eliminates the need for lookup tables
4. The MTPA algorithm helps in achieving the best possible efficiency for given motor operation condition
5. Improved start-up technique is presented, which ensures smooth and stable transition of motor from open loop to close loop
6. Application specific algorithms for washing machine, compressor and fan type loads are presented, along with their results

8 Appendix

8.1 Motor Parameters

8.1.1 Washing Machine (SPMSM)

Pole pairs	12	-
Stator resistance (Rs)	5.2	Ohm
Stator inductance (Ld)	25	mH
Stator inductance (Lq)	25	mH
Voltage constant (Kfi)	465	Vll_pk/KRPM

8.1.2 Air Conditioner Compressor (IPMSM)

Pole pairs	2	-
Stator resistance (Rs)	0.95	Ohm
Stator inductance (Ld)	18.2	mH
Stator inductance (Lq)	31.1	mH
Voltage constant (Kfi)	59.255	Vll_pk/KRPM

8.1.3 Refrigerator Compressor (IPMSM)

Pole pairs	3	-
Stator resistance (Rs)	10	Ohm
Stator inductance (Ld)	46.44	mH
Stator inductance (Lq)	70.5	mH
Voltage constant (Kfi)	110	Vll_pk/KRPM

8.1.4 High Voltage Fan (SPMSM)

Pole pairs	4	-
Stator resistance (Rs)	40.05	Ohm
Stator inductance (Ld)	140	mH
Stator inductance (Lq)	156	mH
Voltage constant (Kfi)	145.45	Vll_pk/KRPM

8.1.5 Low Voltage Fan (SPMSM)

Pole pairs	14	-
Stator resistance (Rs)	0.588	Ohm
Stator inductance (Ld)	1.4773	mH
Stator inductance (Lq)	1.4773	mH
Voltage constant (Kfi)	25.46	Vll_pk/KRPM

8.2 Complete Derivation of Estimator from Motor Model

$$[r_{dqs}] = \begin{bmatrix} r_s & 0 \\ 0 & r_s \end{bmatrix}$$

$$[L_{dqs}] = \begin{bmatrix} L_{ds} & 0 \\ 0 & L_{qs} \end{bmatrix}$$

$$[v_{dqs}] = \begin{bmatrix} v_{ds} \\ v_{qs} \end{bmatrix}$$

$$[i_{dqs}] = \begin{bmatrix} i_{ds} \\ i_{qs} \end{bmatrix}$$

$$[\Psi_{dqs}] = \begin{bmatrix} \Psi_{ds} \\ \Psi_{qs} \end{bmatrix} = [L_{dqs}][i_{dqs}] + \begin{bmatrix} \Psi_{PM} \\ 0 \end{bmatrix} = [L_{dqs}][i_{dqs}] + [\Psi_{dqr}]$$

$$[C] = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \Rightarrow [C]^{-1} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

$$[v_{dqs}] = [r_{dqs}][i_{dqs}] + p[\Psi_{dqs}] + \omega_r \begin{bmatrix} -\Psi_{qs} \\ \Psi_{ds} \end{bmatrix}$$

$$\Rightarrow [v_{dqs}] = [r_{dqs}][i_{dqs}] + p([\Psi_{dqs}]) + \begin{bmatrix} -\omega_r L_{qs} i_{qs} \\ \omega_r L_{ds} i_{ds} \end{bmatrix} + [E_{dqr}]$$

$$\Rightarrow [C]^{-1}[v_{dqs}] = [C]^{-1}[r_{dqs}][i_{dqs}] + [C]^{-1} p([\Psi_{dqs}]) + [C]^{-1} \begin{bmatrix} -\omega_r L_{qs} i_{qs} \\ \omega_r L_{ds} i_{ds} \end{bmatrix} + [C]^{-1}[E_{dqr}]$$

$$\Rightarrow [v_{\alpha\beta s}] = ([C]^{-1}[r_{dqs}][C])([C]^{-1}[i_{dqs}]) + [C]^{-1} p([\Psi_{dqs}]) + [C]^{-1} \begin{bmatrix} -\omega_r L_{qs} i_{qs} \\ \omega_r L_{ds} i_{ds} \end{bmatrix} + [E_{\alpha\beta r}]$$

$$\Rightarrow [v_{\alpha\beta s}] = ([C]^{-1}[r_{dqs}][C])[i_{\alpha\beta s}] + [C]^{-1} p([\Psi_{dqs}]) + [C]^{-1} \begin{bmatrix} -\omega_r L_{qs} i_{qs} \\ \omega_r L_{ds} i_{ds} \end{bmatrix} + [E_{\alpha\beta r}]$$

$$[\Psi_{dqs}]=[L_{dqs}][i_{dqs}]+[\Psi_{dqr}]$$

$$\Rightarrow [\Psi_{dqs}]=[C]\left([C]^{-1}[L_{dqs}][C]\right)\left([C]^{-1}[i_{dqs}]\right)+[\Psi_{dqr}]$$

$$\Rightarrow [\Psi_{dqs}]=[C]\left([C]^{-1}[L_{dqs}][C]\right)[i_{\alpha\beta s}]+[\Psi_{dqr}]$$

$$[C]^{-1}[L_{dqs}][C]:$$

$$[C]^{-1}[L_{dqs}][C]=\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}\begin{bmatrix}L_{ds} & 0\\ 0 & L_{qs}\end{bmatrix}\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}$$

$$=\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}\begin{bmatrix}L_{ds}\cos\theta & L_{ds}\sin\theta\\ -L_{qs}\sin\theta & L_{qs}\cos\theta\end{bmatrix}=$$

$$\begin{bmatrix}L_{ds}\cos^2\theta+L_{qs}\sin^2\theta & L_{ds}\cos\theta\sin\theta-L_{qs}\cos\theta\sin\theta\\ L_{ds}\cos\theta\sin\theta-L_{qs}\cos\theta\sin\theta & L_{ds}\sin^2\theta+L_{qs}\cos^2\theta\end{bmatrix}$$

$$L_{ds}=L_0+L_1;L_{qs}=L_0-L_1$$

$$\Rightarrow [C]^{-1}[L_{dqs}][C]=$$

$$\begin{bmatrix}(L_0+L_1)\cos^2\theta+(L_0-L_1)\sin^2\theta & (L_0+L_1)\cos\theta\sin\theta-(L_0-L_1)\cos\theta\sin\theta\\ (L_0+L_1)\cos\theta\sin\theta-(L_0-L_1)\cos\theta\sin\theta & (L_0+L_1)\sin^2\theta+(L_0-L_1)\cos^2\theta\end{bmatrix}$$

$$=\begin{bmatrix}L_0+L_1\cos2\theta & L_1\sin2\theta\\ L_1\sin2\theta & L_0-L_1\cos2\theta\end{bmatrix}=[L_{\alpha\beta s}]$$

$$\therefore [\Psi_{dqs}]=[C][L_{\alpha\beta s}][i_{\alpha\beta s}]+[\Psi_{dqr}]$$

$$\Rightarrow p\left([\Psi_{dqs}]\right)=p\left([C][L_{\alpha\beta s}][i_{\alpha\beta s}]+[\Psi_{dqr}]\right)$$

$$=p\left([C][L_{\alpha\beta s}][i_{\alpha\beta s}]\right)+p\left([\Psi_{dqr}]\right)$$

$$=[C]p\left([L_{\alpha\beta s}][i_{\alpha\beta s}]\right)+p\left([C]\right)[L_{\alpha\beta s}][i_{\alpha\beta s}]+0$$

$$\therefore [v_{\alpha\beta s}]=[C]^{-1}[r_{dqs}][C][i_{\alpha\beta s}]+[C]^{-1}p\left([\Psi_{dqs}]\right)+[C]^{-1}\begin{bmatrix}-\omega_r L_{qs}i_{qs}\\ \omega_r L_{ds}i_{ds}\end{bmatrix}+[E_{\alpha\beta r}]$$

$$\Rightarrow [v_{\alpha\beta s}]=[r_{dqs}][i_{\alpha\beta s}]+[C]^{-1}\left([C]p\left([L_{\alpha\beta s}][i_{\alpha\beta s}]\right)+p\left([C]\right)[L_{\alpha\beta s}][i_{\alpha\beta s}]\right)+[C]^{-1}\begin{bmatrix}-\omega_r L_{qs}i_{qs}\\ \omega_r L_{ds}i_{ds}\end{bmatrix}+[E_{\alpha\beta r}]$$

$$\Rightarrow [v_{\alpha\beta s}]=[r_{dqs}][i_{\alpha\beta s}]+p\left([L_{\alpha\beta s}][i_{\alpha\beta s}]\right)+[C]^{-1}p\left([C]\right)[L_{\alpha\beta s}][i_{\alpha\beta s}]+[C]^{-1}\begin{bmatrix}-\omega_r L_{qs}i_{qs}\\ \omega_r L_{ds}i_{ds}\end{bmatrix}+[E_{\alpha\beta r}]$$

$$[C]^{-1}p\left([C]\right)=\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}*\omega_r*\begin{bmatrix}-\sin\theta & \cos\theta\\ -\cos\theta & -\sin\theta\end{bmatrix}=\omega_r\begin{bmatrix}0 & 1\\ -1 & 0\end{bmatrix}$$

$$\Rightarrow [C]^{-1}p\left([C]\right)=\omega_r\begin{bmatrix}0 & 1\\ -1 & 0\end{bmatrix}$$

$$[v_{\alpha\beta s}] = [r_{dqs}][i_{\alpha\beta s}] + p([L_{\alpha\beta s}][i_{\alpha\beta s}]) + [C]^{-1} p([C])[L_{\alpha\beta s}][i_{\alpha\beta s}] + [C]^{-1}\begin{bmatrix} -\omega_r L_{qs} i_{qs} \\ \omega_r L_{ds} i_{ds} \end{bmatrix} + [E_{\alpha\beta r}]$$

$$\Rightarrow [v_{\alpha\beta s}] = [r_{dqs}][i_{\alpha\beta s}] + p([L_{\alpha\beta s}][i_{\alpha\beta s}]) + \omega_r \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}[L_{\alpha\beta s}][i_{\alpha\beta s}] + [C]^{-1}\begin{bmatrix} -\omega_r L_{qs} i_{qs} \\ \omega_r L_{ds} i_{ds} \end{bmatrix} + [E_{\alpha\beta r}]$$

$$\omega_r \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}[L_{\alpha\beta s}][i_{\alpha\beta s}] = \omega_r \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} L_0 + L_1 \cos 2\theta & L_1 \sin 2\theta \\ L_1 \sin 2\theta & L_0 - L_1 \cos 2\theta \end{bmatrix}\begin{bmatrix} i_{\alpha s} \\ i_{\beta s} \end{bmatrix}$$

$$= \omega_r \begin{bmatrix} L_1 \sin 2\theta & L_0 - L_1 \cos 2\theta \\ -L_0 - L_1 \cos 2\theta & -L_1 \sin 2\theta \end{bmatrix}\begin{bmatrix} i_{\alpha s} \\ i_{\beta s} \end{bmatrix}$$

$$= \omega_r \begin{bmatrix} L_1 \sin 2\theta i_{\alpha s} + i_{\beta s}(L_0 - L_1 \cos 2\theta) \\ -L_1 \sin 2\theta i_{\beta s} - i_{\alpha s}(L_0 + L_1 \cos 2\theta) \end{bmatrix}$$

$$[C]^{-1}\begin{bmatrix} -\omega_r L_{qs} i_{qs} \\ \omega_r L_{ds} i_{ds} \end{bmatrix} = \omega_r \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} -L_{qs} i_{qs} \\ L_{ds} i_{ds} \end{bmatrix}$$

$$= \omega_r \begin{bmatrix} -L_{qs} i_{qs} \cos\theta - L_{ds} i_{ds} \sin\theta \\ -L_{qs} i_{qs} \sin\theta + L_{ds} i_{ds} \cos\theta \end{bmatrix}$$

$$= \omega_r \begin{bmatrix} -L_{qs}(-i_{\alpha s} \sin\theta + i_{\beta s} \cos\theta)\cos\theta - L_{ds}(i_{\alpha s} \cos\theta + i_{\beta s} \sin\theta)\sin\theta \\ -L_{qs}(-i_{\alpha s} \sin\theta + i_{\beta s} \cos\theta)\sin\theta + L_{ds}(i_{\alpha s} \cos\theta + i_{\beta s} \sin\theta)\cos\theta \end{bmatrix}$$

$$= \omega_r \begin{bmatrix} i_{\alpha s}((L_{qs} - L_{ds})\sin\theta\cos\theta) - i_{\beta s}(L_{ds} \sin^2\theta + L_{qs} \cos^2\theta) \\ i_{\alpha s}(L_{ds} \cos^2\theta + L_{qs} \sin^2\theta) - i_{\beta s}((L_{qs} - L_{ds})\sin\theta\cos\theta) \end{bmatrix}$$

$$= \omega_r \begin{bmatrix} i_{\alpha s}(-2L_1 \sin\theta\cos\theta) - i_{\beta s}((L_0 + L_1)\sin^2\theta + (L_0 - L_1)\cos^2\theta) \\ i_{\alpha s}((L_0 + L_1)\cos^2\theta + (L_0 - L_1)\sin^2\theta) + i_{\beta s}(2L_1 \sin\theta\cos\theta) \end{bmatrix}$$

$$= \omega_r \begin{bmatrix} i_{\alpha s}(-L_1 \sin 2\theta) - i_{\beta s}(L_0 - L_1 \cos 2\theta) \\ i_{\alpha s}(L_0 + L_1 \cos 2\theta) + i_{\beta s}(L_1 \sin 2\theta) \end{bmatrix}$$

$$\therefore \omega_r \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}[L_{\alpha\beta s}][i_{\alpha\beta s}] + [C]^{-1}\begin{bmatrix} -\omega_r L_{qs} i_{qs} \\ \omega_r L_{ds} i_{ds} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\therefore [v_{\alpha\beta s}] = [r_{dqs}][i_{\alpha\beta s}] + p([L_{\alpha\beta s}][i_{\alpha\beta s}]) + [E_{\alpha\beta r}]$$

$$\boxed{[v_{\alpha\beta s}] = [r_{dqs}][i_{\alpha\beta s}] + p([L_{\alpha\beta s}][i_{\alpha\beta s}]) + [E_{\alpha\beta r}]}$$

Where

$$\boxed{\begin{aligned} [L_{\alpha\beta s}] &= \begin{bmatrix} L_0 + L_1 \cos 2\theta & L_1 \sin 2\theta \\ L_1 \sin 2\theta & L_0 - L_1 \cos 2\theta \end{bmatrix} \\ L_0 &= \frac{L_{ds} + L_{qs}}{2} \\ L_1 &= \frac{L_{ds} - L_{qs}}{2} \end{aligned}}$$